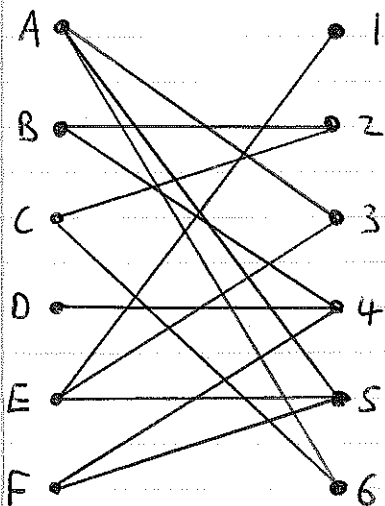
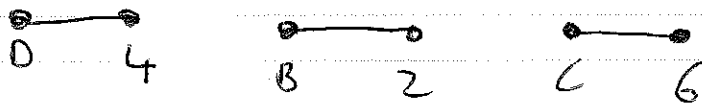
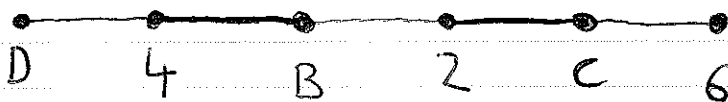


1(a)



(b) Initial Match: A-3 B-4 C-2 E-5

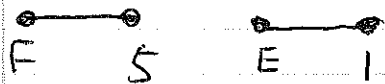
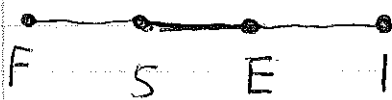
Alt. path:



New match:

- A-3
- B-2
- C-6
- D-4
- ~~E-5~~

Alt. path:



Complete Match:

- A3
- B2
- C6
- D4
- E1
- F5

2(a)

P B M N J K R D  
B M N J K D P R  
B M N J K D P R  
B J K D M N P R  
B D J K M N P R

(b)(i)  $1 + 2 + 3 + 4 + 5 + 6 + 7 = \boxed{28}$

(ii) They must have been in reverse order  
(ie. descending).

3(a)

(i) 10

(ii)  $n - 1$

(b) (i)

$$AB = 18$$

$$BC = 13$$

$$BD = 15$$

$$CF = 16$$

$$DG = 17$$

$$GK = 12$$

$$JK = 16$$

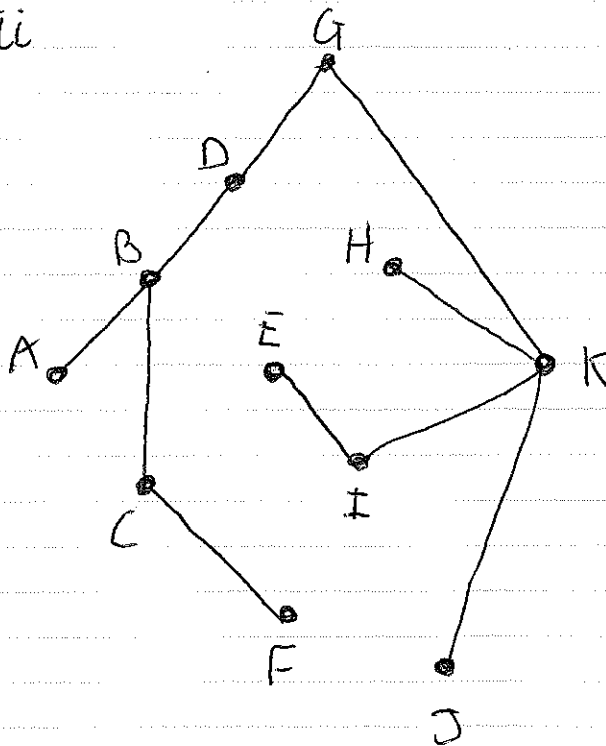
$$KI = 17$$

$$EI = 14$$

$$HK = 17$$

(ii) 155

(iii)



4(a)(i)  $8 + 48 + 18 + 43 + 13 = 130$

(ii)

$$\begin{array}{cccccc} T & \rightarrow & P & \rightarrow & C & \rightarrow & B & \rightarrow & V & \rightarrow & T \\ & & 8 & & 18 & & 43 & & 18 & & 51 \end{array}$$

total = 138

(iii) Nearest Neighbour doesn't give the best solution. The answer above may be improved on.

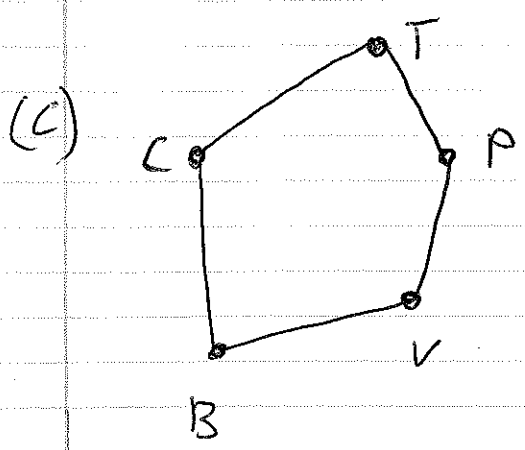
(b)(i) Two shortest from B:  $18 + 43 = \underline{61}$

(Use pairs in table growing B row & column to get spanning tree)

$$\begin{array}{ccc} PT & CT & PV \\ 8 & 13 & 48 \end{array} = 69$$

$61 + 69 = \boxed{130}$   
(Lower bound)

(ii) The solution can't possibly be lower, but the above may not be a tour.



This is a tour.

~~Must be optimum as it is equal to tour found in a(i).~~

Must be optimum as it is equal to tour found in a(i).

- 4 David, a tourist, wishes to visit five places in Rome: Basilica ( $B$ ), Coliseum ( $C$ ), Pantheon ( $P$ ), Trevi Fountain ( $T$ ) and Vatican ( $V$ ). He is to start his tour at one of the places, visit each of the other places, before returning to his starting place.

The table shows the times, in minutes, to travel between these places. David wishes to keep his travelling time to a minimum.

	$B$	$C$	$P$	$T$	$V$
$B$		43	57	52	18
$C$	43		18	13	56
$P$	57	18		8	48
$T$	52	13	8		51
$V$	18	56	48	51	

*Handwritten notes: 3 above C, 2 above T, 4 above V, and a checkmark with 'b(i)' next to the table.*

- (a) (i) Find the total travelling time for the tour  $TPVBCT$ . (1 mark)
- (ii) Find the total travelling time for David's tour using the nearest neighbour algorithm starting from  $T$ . (4 marks)
- (iii) Explain why your answer to part (a)(ii) is an upper bound for David's minimum total travelling time. (2 marks)
- (b) (i) By deleting  $B$ , find a lower bound for the total travelling time for the minimum tour. (5 marks)
- (ii) Explain why your answer to part (b)(i) is a lower bound for David's minimum total travelling time. (2 marks)
- (c) Sketch a network showing the edges that give the lower bound found in part (b)(i) and comment on its significance. (2 marks)

5 (a) Odd vertices:

A B C D

$$AB = 270 \quad (AEFB)$$

$$AC = 290 \quad (AEHGC)$$

$$AD = 260 \quad (AEHD)$$

$$BC = 270 \quad (AFGC)$$

$$BD = 290 \quad (BFED)$$

$$CD = 270 \quad (CGHD)$$

$$AB + CD = 270 + 270 = 540$$

$$AC + BD = 290 + 290 = 580$$

$$AD + BC = 260 + 270 = 530$$

$$1920 + 530 = \underline{2450}$$

repeat AD  
BC

(b) No need to repeat AD. So  $1920 + 270 = \underline{2190}$

(c) (i) Just need to make the graph semi-eulerian.  
So, repeat only AD = 260 (minimum pairing found in A)

$$1920 + 260 = \underline{2180}$$

(ii) Would have to start from one of the two remaining odd vertices  
B or C



6(a)

$$\begin{aligned}
 x &\geq 0 & x &\leq 100 \\
 y &\geq 0 & y &\leq 80 \\
 x &< y \\
 x + y &\geq 60 \\
 2x + 8y &\geq 320
 \end{aligned}$$

Minimise:  $C = 1.5x + 3y$

(b) See figure 1

Note: to find gradient of OL:

$$C = 1.5x + 3y$$

$$\textcircled{3} \quad \frac{C}{3} = \frac{1}{2}x + y$$

$$y = \frac{C}{3} \left[ -\frac{1}{2} \right] x$$

So  $m = -\frac{1}{2}$

(c) ~~Best~~ Solution is at intersection of

$$x + y = 60 \quad \textcircled{1}$$

$$x + 4y = 160 \quad \textcircled{2}$$

$$\textcircled{2} - \textcircled{1}$$

$$3y = 100$$

$$y = 33\frac{1}{3}$$

$$\Rightarrow x = 26\frac{2}{3}$$

P10 →

Consider  
integers

$$26 \leq x \leq 28$$

$$32 \leq y \leq 34$$

Values for  $C$ :

<del>Ans</del>	$x \rightarrow$	26	27	28
$y \downarrow$	32	/	/	/
33	/	/	(141)	
34	(141)	1425	144	

/ = Outside feasible region

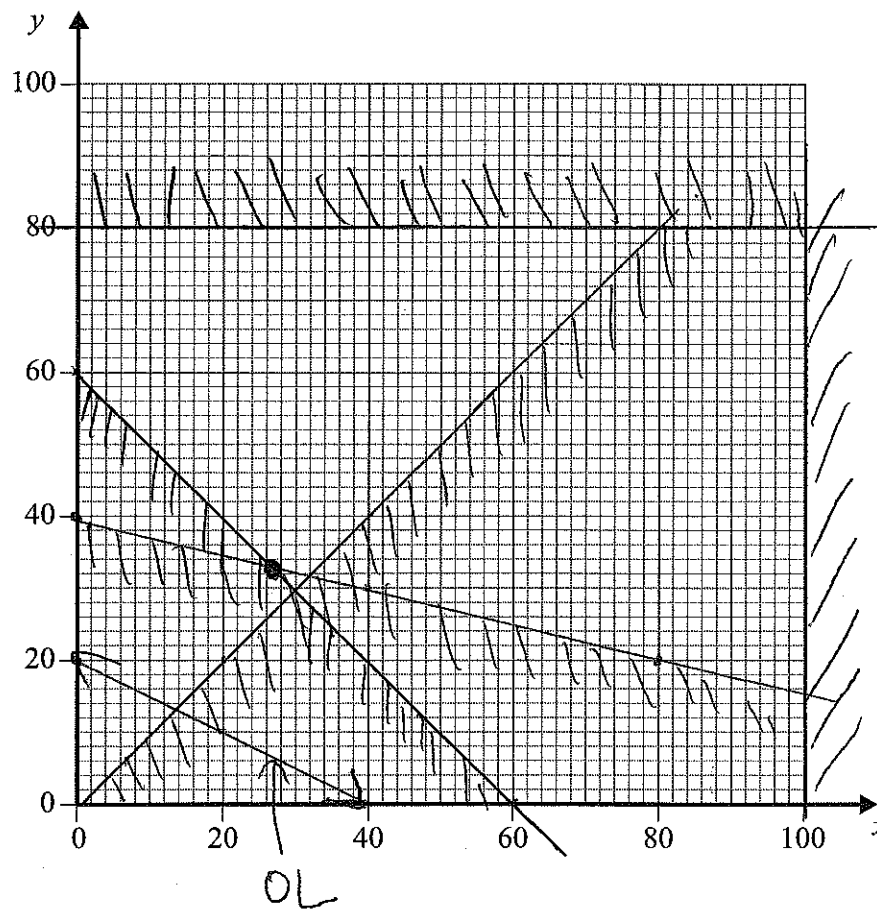
so

$$x = 26 \quad y = 34$$

$$C = 141$$



Figure 1 (for use in Question 6)



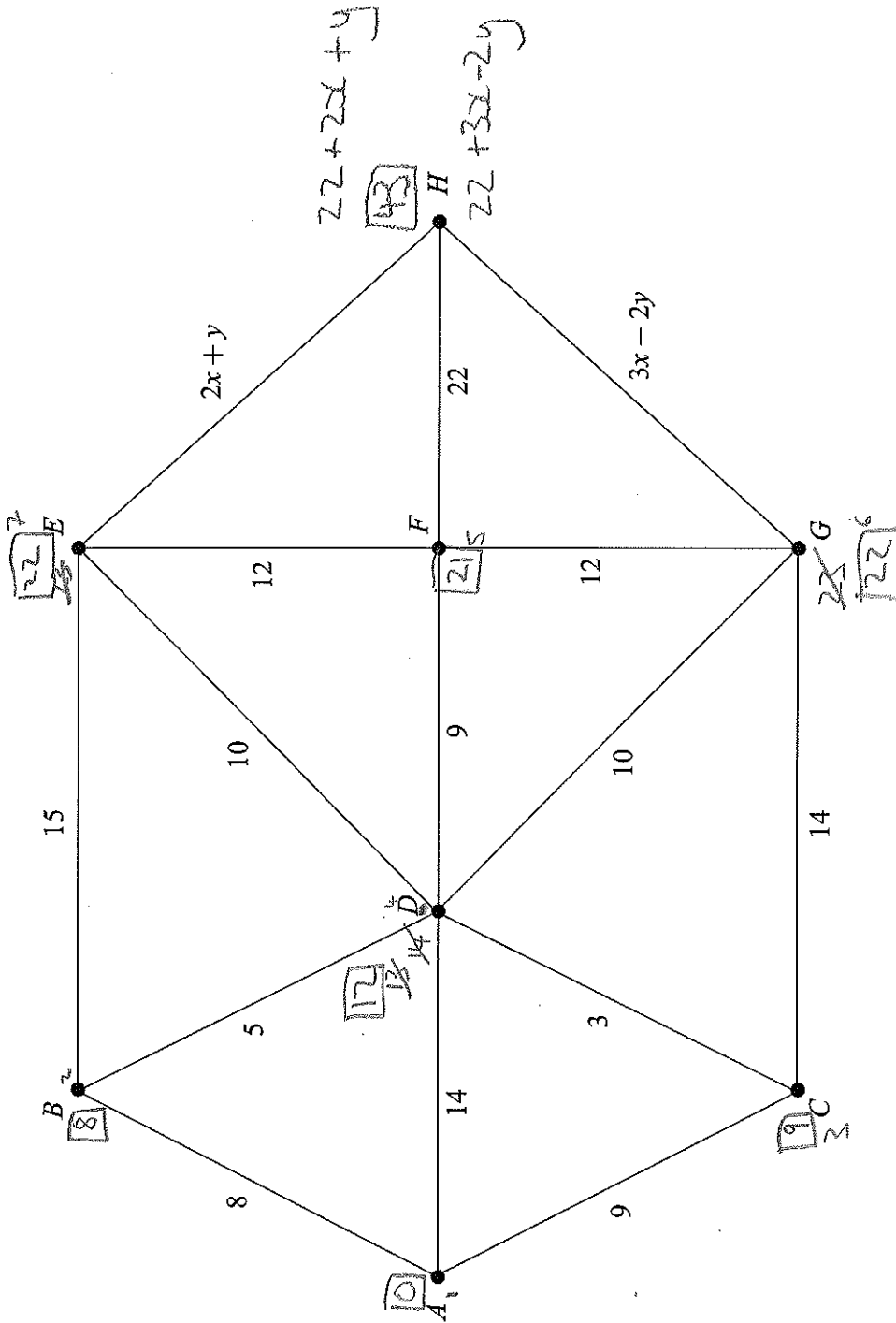
$$2x + 8y \geq 320$$

$$x + 4y \geq 160 \rightarrow \left( \begin{array}{l} \text{consider } x + 4y = 160 \\ y = 40 - \frac{1}{4}x \end{array} \right)$$

$$(0, 40)$$

$$m = \frac{-1}{4} = \frac{-10}{40} \rightarrow \text{down 10 across 40}$$

Figure 2 (for use in Question 7)



$\boxed{7}$

7(a)  $\boxed{43}$

(b)  $22 + 2x + y = 43$   
 $22 + 3x - 2y = 43$

$2x + y = 21$  ①  
 $3x - 2y = 21$  ②

①  $\times 2$

$4x + 2y = 42$   
 $+ 3x - 2y = 21$   


---

 $7x = 63$

$\boxed{x = 9}$

$3 \times 9 - 2y = 21$   
 $27 - 2y = 21$   
 $2y = 6$   $\boxed{y = 3}$